5.1/26

$y = x^2 \ln x$

properties
increasing/decreasing nature of orig function

$y' = [(2x)\ln x] + (x^2)\frac{1}{x} = 2x\ln x + x$

so $y' = x(2\ln x + 1)$

properties
concavity

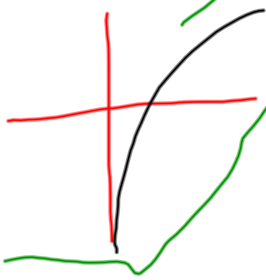
points of inflection

$y'' = [(2)\ln x + (2x)\frac{1}{x}] + 1$

$= 2\ln x + 3$

5.1/26

$\ln x = y$ then $e^y = x$



$y = x^2 \ln x$ [note: $x > 0$]

so $y' = x(2\ln x + 1)$

$y'' = 2\ln x + 3$

if a function changes signs at $x = a$ then $f(a) = 0$ OR infinite discontinuity [i.e. $f(a)$ is not defined]

sign chart of y'

$(+)(2(-2)+1) = -$

$(1)(2\ln 1 + 1) = (1)(1) =$

$- \; - \; | \; + \; + \; + \; + \; +$

$\frac{1}{e^2}$ * $\quad \frac{1}{\sqrt{e}}$ $\quad 1$

$y' = 0$

A) set $f'(x) = 0$ [and solve for x]

domain of $\ln x = (0, \infty)$

$x(2\ln x + 1) = 0$

$\hookrightarrow 2\ln x + 1 = 0$ i.e. $\ln x = \frac{-1}{2}$, $x = e^{-\frac{1}{2}}$

$\hookrightarrow x = 0$

B) find domain of y'

* $\frac{1}{e^2} = e^{-2}$

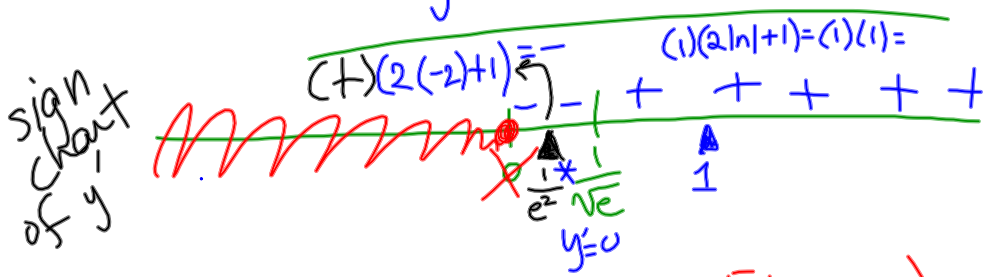
5.1/26

$y = x^2 \ln x$ [note: $x>0$]

so $y' = x(2\ln x + 1)$

$y'' = 2\ln x + 3$

if a function changes signs at $x = a$ then $f(a) = 0$ OR infinite discontinuity [i.e. $f(a)$ is not defined]



26a) $f(x)$ is increasing on $\left[\frac{1}{\sqrt{e}}, \infty\right)$

26b) $f(x)$ is decreasing on $\left(0, \frac{1}{\sqrt{e}}\right]$

* $\frac{1}{e^2} = e^{-2}$

5.1/26

$y = x^2 \ln x$ [note: $x > 0$]

so $y' = x(2\ln x + 1)$

$y'' = 2\ln x + 3$

if a function changes signs at $x = a$ then $f(a) = 0$ OR infinite discontinuity [i.e. $f(a)$ is not defined]

Sign chart of y''

$y'' = 2\ln(e^{-10}) + 3 = -$ $y'' = 2\ln 1 + 3 = +$

$- \; - \; | \; + \; + \; +$

$0 \quad e^{-10} \quad \sqrt{e^3} \quad 1$

A) set $y'' = 0$ & solve

$2\ln x + 3 = 0 \Rightarrow \ln x = -\frac{3}{2}$

so $x = e^{-\frac{3}{2}} = \frac{1}{\sqrt{e^3}}$

B) check for undefined [domain]

$x > 0$ this is continuous

* $\frac{1}{e^2} = e^{-2}$

5.1/26

$y = x^2 \ln x$ [note: $x > 0$]

so $y' = x(2\ln x + 1)$

$y'' = 2\ln x + 3$

if a function changes signs at $x = a$ then $f(a) = 0$ OR infinite discontinuity [i.e. $f(a)$ is not defined]

Sign chart of y''

$y'' = 2\ln(e^{-10}) + 3 =$ − − − $y'' = 2\ln 1 + 3 =$ + + +

$0 \quad e^{-10} \quad \frac{1}{\sqrt{e^3}} \quad 1$

* $\frac{1}{e^2} = e^{-2}$

26c) graph of $f(x)$ is concave up when $f'' > 0$: $\left(\frac{1}{\sqrt{e^3}}, \infty\right)$

26d) graph of $f(x)$ is concave down when $f'' < 0$: $\left(0, \frac{1}{\sqrt{e^3}}\right)$

26(e) $x = \frac{1}{\sqrt{e^3}}$ is the x value of the only point of inflection

y coordinate is $\left(\frac{1}{\sqrt{e^3}}\right)^2 \ln\left(\frac{1}{\sqrt{e^3}}\right) = \frac{1}{e^3}\left(\frac{-3}{2}\right)$

Practice Consider $y = ax^2 + bx + C$

a) where is y increasing?

b) where is y decreasing?

c) where is y concave up?

d) where is y concave down?

e) where are points of inflection?

$y' = 2ax + b$

B) always defined

A) $2ax + b = 0$

$x = \frac{-b}{2a}$

$a > 0$

$- - - \mid + + +$ at $\frac{-b}{2a}$

$a < 0$

$+ + + + \mid - - -$ at $\frac{-b}{2a}$

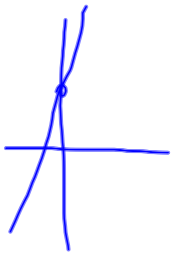
$y'' = 2a$

$a > 0$ $+ + + + + + +$

$a < 0$ $- - - - -$

Z → 3.7/1

(2) $y = 3x + 5$



(3) $\frac{dy}{dt} = 3\frac{dx}{dt}$

(4) a) $\frac{dx}{dt} = 2$; $x = 1$

find $\frac{dy}{dt}$.

$\boxed{\frac{dy}{dt} = 3(2) = 6}$

b) $\frac{dy}{dt} = -1$; $x = 0$

$(-1) = 3\frac{dx}{dt}$ so $\boxed{\frac{dx}{dt} = \frac{-1}{3}}$

$g(x) = 3$

$g(2) = 3$

$g(17) = 3$

$\vdots$

1) picture

2) equation true for every time t

3) derivative-equation true for every time t

4) substitute information for a specific t

5) conclusion/solve...

remember!

→ $x = x(t)$, $y = g(t)$

→ chain rule

$\Rightarrow$ 3.7 / 5

1

2 $A = x^2$

3 $\frac{dA}{dt} = 2x\frac{dx}{dt}$

4 $x = 3^{ft}$; $\frac{dx}{dt} = 2\,\frac{ft}{min}$

$$\frac{dA}{dt} = 2(\overset{x}{3})(\overset{\frac{dx}{dt}}{2}) = 12\,\frac{ft^2}{min}$$